Academic Standards for Mathematics

Pennsylvania Department of Education

Academic Standards for Mathematics

IV. TABLE OF CONTENTS

Academic Standards for Mathematics

V. INTRODUCTION

This document includes Mathematics Standards:

- ◊ 2.1. Numbers, Number Systems and Number Relationships
- ◊ 2.2. Computation and Estimation
- ◊ 2.3. Measurement and Estimation
- ◊ 2.4. Mathematical Reasoning and Connections
- ◊ 2.5. Mathematical Problem Solving and Communication
- ◊ 2.6. Statistics and Data Analysis
- ◊ 2.7. Probability and Predictions
- ◊ 2.8. Algebra and Functions
- ◊ 2.9. Geometry
- ◊ 2.10. Trigonometry
- ◊ 2.11. Concepts of Calculus

The Mathematics Standards describe what students should know and be able to do at four grade levels (third, fifth, eighth and eleventh). They reflect the increasing complexity and sophistication that students are expected to achieve as they progress through school.

This document avoids repetition of learned skills, making an obvious progression across grade levels less explicit. Teachers shall expect that students know and can apply the concepts and skills expressed at the preceding level. Consequently, previous learning is reinforced but not retaught.

Students who achieve these mathematical standards will be able to communicate mathematically. Although it is an interesting and enjoyable study for its own sake, mathematics is most appropriately used as a tool to help organize and understand information from other academic disciplines. Because our capacity to deal with all things mathematical is changing rapidly, students must be able to bring the most modern and effective technology to bear on their learning of mathematical concepts and skills.

A glossary is included to assist the reader in understanding terminology contained in the standards.

Academic Standards for Mathematics

2.1. Numbers, Number Systems and Number Relationships			
2.1.3. **GRADE 3**	2.1.5. **GRADE 5**	2.1.8. **GRADE 8**	2.1.11. **GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:			
A. Count using whole numbers (to 10,000) and by 2's, 3's, 5's, 10's, 25's and 100's.	A. Use expanded notation to represent whole numbers or decimals.	A. Represent and use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, exponents, scientific notation, square roots).	A. Use operations (e.g., opposite, reciprocal, absolute value, raising to a power, finding roots, finding logarithms).
B. Use whole numbers and fractions to represent quantities.	B. Apply number theory concepts to rename a number quantity (e.g., six, 6, $\frac{12}{2}$, 3×2, $10 - 4$).	B. Simplify numerical expressions involving exponents, scientific notation and using order of operations.	
C. Represent equivalent forms of the same number through the use of concrete objects, drawings, word names and symbols.	C. Demonstrate that mathematical operations can represent a variety of problem situations.	C. Distinguish between and order rational and irrational numbers.	
D. Use drawings, diagrams or models to show the concept of fraction as part of a whole.	D. Use models to represent fractions and decimals.	D. Apply ratio and proportion to mathematical problem situations involving distance, rate, time and similar triangles.	
E. Count, compare and make change using a collection of coins and one-dollar bills.	E. Explain the concepts of prime and composite numbers.	E. Simplify and expand algebraic expressions using exponential forms.	
F. Apply number patterns (even and odd) and compare values of numbers on the hundred board.	F. Use simple concepts of negative numbers (e.g., on a number line, in counting, in temperature).	F. Use the number line model to demonstrate integers and their applications.	

G. Use concrete objects to count, order and group. H. Demonstrate an understanding of one-to-one correspondence. I. Apply place-value concepts and numeration to counting, ordering and grouping. J. Estimate, approximate, round or use exact numbers as appropriate. K. Describe the inverse relationship between addition and subtraction. L. Demonstrate knowledge of basic facts in four basic operations.	G. Develop and apply number theory concepts (e.g., primes, factors, multiples, composites) to represent numbers in various ways.	G. Use the inverse relationships between addition, subtraction, multiplication, division, exponentiation and root extraction to determine unknown quantities in equations.	

2.2. Computation and Estimation

Academic Standards for Mathematics

2.2.3. **GRADE 3**	2.2.5. **GRADE 5**	2.2.8. **GRADE 8**	2.2.11. **GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:			
A. Apply addition and subtraction in everyday situations using concrete objects.	A. Create and solve word problems involving addition, subtraction, multiplication and division of whole numbers.	A. Complete calculations by applying the order of operations.	A. Develop and use computation concepts, operations and procedures with real numbers in problem-solving situations.
B. Solve single- and double-digit addition and subtraction problems with regrouping in vertical form.	B. Develop and apply algorithms to solve word problems that involve addition, subtraction, and/or multiplication with decimals with and without regrouping.	B. Add, subtract, multiply and divide different kinds and forms of rational numbers including integers, decimal fractions, percents and proper and improper fractions.	B. Use estimation to solve problems for which an exact answer is not needed.
C. Demonstrate the concept of multiplication as repeated addition and arrays.	C. Develop and apply algorithms to solve word problems that involve addition, subtraction, and/or multiplication with fractions and mixed numbers that include like and unlike denominators.	C. Estimate the value of irrational numbers.	C. Construct and apply mathematical models, including lines and curves of best fit, to estimate values of related quantities.
D. Demonstrate the concept of division as repeated subtraction and as sharing.	D. Demonstrate the ability to round numbers.	D. Estimate amount of tips and discounts using ratios, proportions and percents.	D. Describe and explain the amount of error that may exist in a computation using estimates.
E. Use estimation skills to arrive at conclusions.	E. Determine through estimations the reasonableness of answers to problems involving addition, subtraction, multiplication and division of whole numbers.	E. Determine the appropriateness of overestimating or underestimating in computation.	E. Recognize that the degree of precision needed in calculating a number depends on how the results will be used and the instruments used to generate the measure.
F. Determine the reasonableness of calculated answers.	F. Demonstrate skills for using fraction calculators to verify conjectures,	F. Identify the difference between exact value and approximation and determine	F. Demonstrate skills for using computer spreadsheets and scientific and

G. Explain addition and subtraction algorithms with regrouping.	confirm computations and explore complex problem-solving situations. G. Apply estimation strategies to a variety of problems including time and money. H. Explain multiplication and division algorithms. I. Select a method for computation and explain why it is appropriate.	which is appropriate for a given situation.	graphing calculators.

Academic Standards for Mathematics

2.3. Measurement and Estimation			
2.3.3. **GRADE 3**	2.3.5. **GRADE 5**	2.3.8. **GRADE 8**	2.3.11. **GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills to:			
A. Compare measurable characteristics of different objects on the same dimensions (e.g., time, temperature, area, length, weight, capacity, perimeter).	A. Select and use appropriate instruments and units for measuring quantities (e.g., perimeter, volume, area, weight, time, temperature).	A. Develop formulas and procedures for determining measurements (e.g., area, volume, distance).	A. Select and use appropriate units and tools to measure to the degree of accuracy required in particular measurement situations.
B. Determine the measurement of objects with non-standard and standard units (e.g., US customary and metric).	B. Select and use standard tools to measure the size of figures with specified accuracy, including length, width, perimeter and area.	B. Solve rate problems (e.g., rate × time = distance, principal × interest rate = interest).	B. Measure and compare angles in degrees and radians.
C. Determine and compare elapsed times.	C. Estimate, refine and verify specified measurements of objects.	C. Measure angles in degrees and determine relations of angles.	C. Demonstrate the ability to produce measures with specified levels of precision.
D. Tell time (analog and digital) to the minute.	D. Convert linear measurements within the same system.	D. Estimate, use and describe measures of distance, rate, perimeter, area, volume, weight, mass and angles.	
E. Determine the appropriate unit of measure.	E. Add and subtract measurements.	E. Describe how a change in linear dimension of an object affects its perimeter, area and volume.	
F. Use concrete objects to determine area and perimeter.		F. Use scale measurements to interpret maps or drawings.	
G. Estimate and verify measurements.		G. Create and use scale models.	
H. Demonstrate that a single object has different attributes that can be measured in different ways (e.g., length, mass,			

weight, time, area, temperature, capacity, perimeter).			

Academic Standards for Mathematics

2.4. Mathematical Reasoning and Connections			
2.4.3. **GRADE 3**	2.4.5. **GRADE 5**	2.4.8. **GRADE 8**	2.4.11. **GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills to:			
A. Make, check and verify predictions about the quantity, size and shape of objects and groups of objects.	A. Compare quantities and magnitudes of numbers.	A. Make conjectures based on logical reasoning and test conjectures by using counter-examples.	A. Use direct proofs, indirect proofs or proof by contradiction to validate conjectures.
B. Use measurements in everyday situations (e.g., determine the geography of the school building).	B. Use models, number facts, properties and relationships to check and verify predictions and explain reasoning.	B. Combine numeric relationships to arrive at a conclusion.	B. Construct valid arguments from stated facts.
	C. Draw inductive and deductive conclusions within mathematical contexts.	C. Use if...then statements to construct simple, valid arguments.	C. Determine the validity of an argument.
	D. Distinguish between relevant and irrelevant information in a mathematical problem.	D. Construct, use and explain algorithmic procedures for computing and estimating with whole numbers, fractions, decimals and integers.	D. Use truth tables to reveal the logic of mathematical statements.
	E. Interpret statements made with precise language of logic (e.g., “all”, “or”, “every”, “none”, “some”, “or”, “many”).	E. Distinguish between inductive and deductive reasoning.	E. Demonstrate mathematical solutions to problems (e.g., in the physical sciences).
	F. Use statistics to quantify issues (e.g., in social studies, in science).	F. Use measurements and statistics to quantify issues (e.g., in family, consumer science situations).	

Academic Standards for Mathematics

2.5. Mathematical Problem Solving and Communication			
2.5.3. **GRADE 3**	2.5.5. **GRADE 5**	2.5.8. **GRADE 8**	2.5.11. **GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills to:			
A. Use appropriate problem-solving strategies (e.g., guess and check, working backwards).	A. Develop a plan to analyze a problem, identify the information needed to solve the problem, carry out the plan, check whether an answer makes sense and explain how the problem was solved.	A. Invent, select, use and justify the appropriate methods, materials and strategies to solve problems.	A. Select and use appropriate mathematical concepts and techniques from different areas of mathematics and apply them to solving non-routine and multi-step problems.
B. Determine when sufficient information is present to solve a problem and explain how to solve a problem.	B. Use appropriate mathematical terms, vocabulary, language symbols and graphs to explain clearly and logically solutions to problems.	B. Verify and interpret results using precise mathematical language, notation and representations, including numerical tables and equations, simple algebraic equations and formulas, charts, graphs and diagrams.	B. Use symbols, mathematical terminology, standard notation, mathematical rules, graphing and other types of mathematical representations to communicate observations, predictions, concepts, procedures, generalizations, ideas and results.
C. Select and use an appropriate method, materials and strategy to solve problems, including mental mathematics, paper and pencil and concrete objects.	C. Show ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams and models. D. Connect, extend and generalize problem solutions to other concepts, problems and circumstances in mathematics. E. Select, use and justify the methods, materials and strategies used to solve problems. F. Use appropriate problem-solving strategies (e.g., solving a simpler problem, drawing a picture or diagram).	C. Justify strategies and defend approaches used and conclusions reached. D. Determine pertinent information in problem situations and whether any further information is needed for solution.	C. Present mathematical procedures and results clearly, systematically, succinctly and correctly. D. Conclude a solution process with a summary of results and evaluate the degree to which the results obtained represent an acceptable response to the initial problem and why the reasoning is valid.

Academic Standards for Mathematics

Academic Standards for Mathematics

2.6. Statistics and Data Analysis			
2.6.3. **GRADE 3**	2.6.5. **GRADE 5**	2.6.8. **GRADE 8**	2.6.11. **GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills to:			
A. Gather, organize and display data using pictures, tallies, charts, bar graphs and pictographs.	A. Organize and display data using pictures, tallies, tables, charts, bar graphs and circle graphs.	A. Compare and contrast different plots of data using values of mean, median, mode, quartiles and range.	A. Design and conduct an experiment using random sampling. Describe the data as an example of a distribution using statistical measures of center and spread. Organize and represent the results with graphs. (Use standard deviation, variance and t-tests.)
B. Formulate and answer questions based on data shown on graphs.	B. Describe data sets using mean, median, mode and range.	B. Explain effects of sampling procedures and missing or incorrect information on reliability.	B. Use appropriate technology to organize and analyze data taken from the local community.
C. Predict the likely number of times a condition will occur based on analyzed data.	C. Sort data using Venn diagrams.	C. Fit a line to the scatter plot of two quantities and describe any correlation of the variables.	C. Determine the regression equation of best fit (e.g., linear, quadratic, exponential).
D. Form and justify an opinion on whether a given statement is reasonable based on a comparison to data.	D. Predict the likely number of times a condition will occur based on analyzed data.	D. Design and carry out a random sampling procedure.	D. Make predictions using interpolation, extrapolation, regression and estimation using technology to verify them.
	E. Construct and defend simple conclusions based on data.	E. Analyze and display data in stem-and-leaf and box-and-whisker plots.	E. Determine the validity of the sampling method described in a given study.
		F. Use scientific and graphing calculators and computer spreadsheets to organize and analyze data.	F. Determine the degree of dependence of two quantities specified by a two-way table.

		G. Determine the validity of the sampling method described in studies published in local or national newspapers.	G. Describe questions of experimental design, control groups, treatment groups, cluster sampling and reliability. H. Use sampling techniques to draw inferences about large populations. I. Describe the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed.

2.7. Probability and Predictions

Academic Standards for Mathematics

2.7.3. **GRADE 3**	2.7.5. **GRADE 5**	2.7.8. **GRADE 8**	2.7.11. **GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:			
A. Predict and measure the likelihood of events and recognize that the results of an experiment may not match predicted outcomes.	A. Perform simulations with concrete devices (e.g., dice, spinner) to predict the chance of an event occurring.	A. Determine the number of combinations and permutations for an event.	A. Compare odds and probability.
B. Design a fair and an unfair spinner.	B. Determine the fairness of the design of a spinner.	B. Present the results of an experiment using visual representations (e.g., tables, charts, graphs).	B. Apply probability and statistics to perform an experiment involving a sample and generalize its results to the entire population.
C. List or graph the possible results of an experiment.	C. Express probabilities as fractions and decimals.	C. Analyze predictions (e.g., election polls).	C. Draw and justify a conclusion regarding the validity of a probability or statistical argument.
D. Analyze data using the concepts of largest, smallest, most often, least often and middle.	D. Compare predictions based on theoretical probability and experimental results.	D. Compare and contrast results from observations and mathematical models.	D. Use experimental and theoretical probability distributions to make judgments about the likelihood of various outcomes in uncertain situations.
	E. Calculate the probability of a simple event. F. Determine patterns generated as a result of an experiment. G. Determine the probability of an event involving “and”, “or” or “not”. H. Predict and determine why some outcomes are certain, more likely, less likely, equally likely or impossible.	E. Make valid inferences, predictions and arguments based on probability.	E. Solve problems involving independent simple and compound events.

	I. Find all possible combinations and arrangements involving a limited number of variables. J. Develop a tree diagram and list the elements.		

2.8. Algebra and Functions			

Academic Standards for Mathematics

2.8.3. **GRADE 3**	2.8.5. **GRADE 5**	2.8.8. **GRADE 8**	2.8.11. **GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills to:			
A. Recognize, describe, extend, create and replicate a variety of patterns including attribute, activity, number and geometric patterns.	A. Recognize, reproduce, extend, create and describe patterns, sequences and relationships verbally, numerically, symbolically and graphically, using a variety of materials.	A. Apply simple algebraic patterns to basic number theory and to spatial relations	A. Analyze a given set of data for the existence of a pattern and represent the pattern algebraically and graphically.
B. Use concrete objects and trial and error to solve number sentences and check if solutions are sensible and accurate.	B. Connect patterns to geometric relations and basic number skills.	B. Discover, describe and generalize patterns, including linear, exponential and simple quadratic relationships.	B. Give examples of patterns that occur in data from other disciplines.
C. Substitute a missing addend in a number sentence.	C. Form rules based on patterns (e.g., an equation that relates pairs in a sequence).	C. Create and interpret expressions, equations or inequalities that model problem situations.	C. Use patterns, sequences and series to solve routine and non-routine problems.
D. Create a story to match a given combination of symbols and numbers.	D. Use concrete objects and combinations of symbols and numbers to create expressions that model mathematical situations.	D. Use concrete objects to model algebraic concepts.	D. Formulate expressions, equations, inequalities, systems of equations, systems of inequalities and matrices to model routine and non-routine problem situations.
E. Use concrete objects and symbols to model the concepts of variables, expressions, equations and inequalities.	E. Explain the use of combinations of symbols and numbers in expressions, equations and inequalities.	E. Select and use a strategy to solve an equation or inequality, explain the solution and check the solution for accuracy.	E. Use equations to represent curves (e.g., lines, circles, ellipses, parabolas, hyperbolas).
F. Explain the meaning of solutions and symbols.	F. Describe a realistic situation using information given in equations, inequalities, tables or graphs.	F. Solve and graph equations and inequalities using scientific and graphing calculators and computer spreadsheets.	F. Identify whether systems of equations and inequalities are consistent or inconsistent.
G. Use a table or a chart to display information.	G. Select and use appropriate strategies, including concrete materials, to solve number sentences and explain the	G. Represent relationships with tables or graphs in the coordinate plane and verbal or symbolic rules.	G. Analyze and explain systems of equations, systems of inequalities and matrices.

Academic Standards for Mathematics

	method of solution.		
H. Describe and interpret the data shown in tables and charts.	H. Locate and identify points on a coordinate system.	H. Graph a linear function from a rule or table.	H. Select and use an appropriate strategy to solve systems of equations and inequalities using graphing calculators, symbol manipulators, spreadsheets and other software.
I. Demonstrate simple function rules.	I. Generate functions from tables of data and relate data to corresponding graphs and functions.	I. Generate a table or graph from a function and use graphing calculators and computer spreadsheets to graph and analyze functions.	I. Use matrices to organize and manipulate data, including matrix addition, subtraction, multiplication and scalar multiplication.
J. Analyze simple functions and relationships and locate points on a simple grid.		J. Show that an equality relationship between two quantities remains the same as long as the same change is made to both quantities; explain how a change in one quantity determines another quantity in a functional relationship.	J. Demonstrate the connection between algebraic equations and inequalities and the geometry of relations in the coordinate plane. K. Select, justify and apply an appropriate technique to graph a linear function in two variables, including slope-intercept, x- and y-intercepts, graphing by transformations and the use of a graphing calculator. L. Write the equation of a line when given the graph of the line, two points on the line, or the slope of the line and a point on the line. M. Given a set of data points, write an equation for a line of best fit. N. Solve linear, quadratic and exponential equations both symbolically and graphically. O. Determine the domain and range of a

			relation, given a graph or set of ordered pairs. P. Analyze a relation to determine whether a direct or inverse variation exists and represent it algebraically and graphically. Q. Represent functional relationships in tables, charts and graphs. R. Create and interpret functional models. S. Analyze properties and relationships of functions (e.g., linear, polynomial, rational, trigonometric, exponential, logarithmic). T. Analyze and categorize functions by their characteristics.

2.9. Geometry			
2.9.3. **GRADE 3**	2.9.5. **GRADE 5**	2.9.8. **GRADE 8**	2.9.11. **GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge			

Academic Standards for Mathematics

and skills to:			
A. Name and label geometric shapes in two and three dimensions (e.g., circle/sphere, square/cube, triangle/pyramid, rectangle/prism).	A. Give formal definitions of geometric figures.	A. Construct figures incorporating perpendicular and parallel lines, the perpendicular bisector of a line segment and an angle bisector using computer software.	A. Construct geometric figures using dynamic geometry tools (e.g., Geometer's Sketchpad, Cabri Geometre).
B. Build geometric shapes using concrete objects (e.g., manipulatives).	B. Classify and compare triangles and quadrilaterals according to sides or angles.	B. Draw, label, measure and list the properties of complementary, supplementary and vertical angles.	B. Prove that two triangles or two polygons are congruent or similar using algebraic, coordinate and deductive proofs.
C. Draw two- and three-dimensional geometric shapes and construct rectangles, squares and triangles on the geoboard and on graph paper satisfying specific criteria.	C. Identify and measure circles, their diameters and their radii.	C. Classify familiar polygons as regular or irregular up to a decagon.	C. Identify and prove the properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles and diagonals using deductive proofs.
D. Find and describe geometric figures in real life.	D. Describe in words how geometric shapes are constructed.	D. Identify, name, draw and list all properties of squares, cubes, pyramids, parallelograms, quadrilaterals, trapezoids, polygons, rectangles, rhombi, circles, spheres, triangles, prisms and cylinders.	D. Identify corresponding parts in congruent triangles to solve problems.
E. Identify and draw lines of symmetry in geometric figures.	E. Construct two- and three-dimensional shapes and figures using manipulatives, geoboards and computer software.	E. Construct parallel lines, draw a transversal and measure and compare angles formed (e.g., alternate interior and exterior angles).	E. Solve problems involving inscribed and circumscribed polygons.
F. Identify symmetry in nature.	F. Find familiar solids in the environment and describe them.	F. Distinguish between similar and congruent polygons.	F. Use the properties of angles, arcs, chords, tangents and secants to solve problems involving circles.
G. Fold paper to demonstrate the reflections about a line.	G. Create an original tessellation.	G. Approximate the value of π (pi) through experimentation.	G. Solve problems using analytic geometry.

H. Show relationships between and among figures using reflections. I. Predict how shapes can be changed by combining or dividing them.	H. Describe the relationship between the perimeter and area of triangles, quadrilaterals and circles. I. Represent and use the concepts of line, point and plane. J. Define the basic properties of squares, pyramids, parallelograms, quadrilaterals, trapezoids, polygons, rectangles, rhombi, circles, triangles, cubes, prisms, spheres and cylinders. K. Analyze simple transformations of geometric figures and rotations of line segments. L. Identify properties of geometric figures (e.g., parallel, perpendicular, similar, congruent, symmetrical).	H. Use simple geometric figures (e.g., triangles, squares) to create, through rotation, transformational figures in three dimensions. I. Generate transformations using computer software. J. Analyze geometric patterns (e.g., tessellations, sequences of shapes) and develop descriptions of the patterns. K. Analyze objects to determine whether they illustrate tessellations, symmetry, congruence, similarity and scale.	H. Construct a geometric figure and its image using various transformations. I. Model situations geometrically to formulate and solve problems. J. Analyze figures in terms of the kinds of symmetries they have.

2.10. Trigonometry			
2.10.3. **GRADE 3**	2.10.5. **GRADE 5**	2.10.8. **GRADE 8**	2.10.11. **GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge			

and skills to:			
A. Identify right angles in the environment.	A. Identify and compare parts of right triangles, including right angles, acute angles, hypotenuses and legs.	A. Compute measures of sides and angles using proportions, the Pythagorean Theorem and right triangle relationships.	A. Use graphing calculators to display periodic and circular functions; describe properties of the graphs.
B. Model right angles and right triangles using concrete objects.	B. Create right triangles on a geoboard.	B. Solve problems requiring indirect measurement for lengths of sides of triangles.	B. Identify, create and solve practical problems involving right triangles using the trigonometric functions and the Pythagorean Theorem.

Academic Standards for Mathematics

2.11. Concepts of Calculus

2.11.3. **GRADE 3**	2.11.5. **GRADE 5**	2.11.8. **GRADE 8**	2.11.11. **GRADE 11**
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills to:			
A. Identify whole number quantities and measurements from least to most and greatest value. B. Identify least and greatest values represented in bar graphs and pictographs. C. Categorize rates of change as faster and slower. D. Continue a pattern of numbers or objects that could be extended infinitely.	A. Make comparisons of numbers (e.g., more, less, same, least, most, greater than, less than). B. Identify least and greatest values represented in bar and circle graphs. C. Identify maximum and minimum. D. Describe the relationship between rates of change and time. E. Estimate areas and volumes as the sums of areas of tiles and volumes of cubes. F. Describe the relationship between the size of the unit of measurement and the estimate of the areas and volumes.	A. Analyze graphs of related quantities for minimum and maximum values and justify the findings. B. Describe the concept of unit rate, ratio and slope in the context of rate of change. C. Continue a pattern of numbers or objects that could be extended infinitely.	A. Determine maximum and minimum values of a function over a specified interval. B. Interpret maximum and minimum values in problem situations. C. Graph and interpret rates of growth/decay. D. Determine sums of finite sequences of numbers and infinite geometric series. E. Estimate areas under curves using sequences of areas.

Academic Standards for Mathematics

VI. GLOSSARY

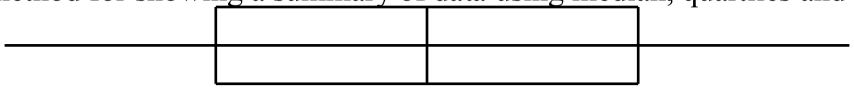
Absolute value: A number's distance from zero on a number line. The absolute value of 2 is equal to the absolute value of -2.

Algorithm: A method of performing an arithmetic operation.

Analog time: Time displayed on a timepiece having hour and minute hands.

Array: Arrangement of a series of items according to the values of the items (e.g., largest to smallest).

Box-and-whisker plot: A graphic method for showing a summary of data using median, quartiles and extremes of data.



Combination: A subset of the elements in a given set, without regard to the order in which those elements are arranged.

Composite number: Any positive integer exactly divisible by one or more positive integers other than itself and 1.

Congruent: Having the same shape and the same size.

Conjecture: A statement believed to be true but not proved.

Academic Standards for Mathematics

Coordinate system: A method of locating points in the plane or in space by means of numbers. A point in the plane is located by its distances from both a horizontal and a vertical line called the axes. The horizontal line is called the x-axis. The vertical line is called the y-axis. The pairs of numbers are called ordered pairs. The first number, called the x-coordinate, designates the distance along the horizontal axis. The second number, called the y-coordinate, designates the distance along the vertical axis. The point at which the two axes intersect has the coordinates (0,0) and is called the origin.

Correlation: A measure of the mutual relationship between two variables.

Customary system: A system of weights and measures frequently used in the United States. The basic unit of weight is the pound; the basic unit of capacity is the quart.

Deductive reasoning: The process of reasoning from statements accepted as true to reach a conclusion.

Direct variation: Two variables are so related that their ratio remains constant.

Domain: The set of all possible values for the unknown in an open sentence.

Equation: A statement of equality between two mathematical expressions (e.g., X + 5 = Y - 2).

Equivalent forms: Different forms of numbers that name the same number (e.g., fraction, decimal, percent as $\frac{1}{2}$, .5, 50%).

Expanded notation: Involves writing the number in expanded form to show the value of each digit (e.g., 15,629 = 10,000 + 5,000 + 600 + 20 + 9).

Exponential function: A function whose general equation is $y = a \times b^x$ or $y = a \times b^{kx}$, where a, b and k stand for constants.

Exponent: A numeral used to tell how many times a number or variable is used as a factor (e.g., a^2, 2^n, y^x).

Expression: A mathematical phrase that can include operations, numerals and variables. In algebraic terms: $2l + 3x$;

	in numeric terms: 13.4 - 4.7.
Factor:	The number or variable multiplied in a multiplication expression.
Factorial:	The expression n! (n factorial) is the product of all the numbers from 1 to n for any positive integer n.
Function:	A relation in which each value of an independent variable is associated with a unique value of the dependent value.
Geoboard:	A board with pegs aligned in grid fashion that permits rubber bands to be wrapped around pegs to form geometric figures.
Graphing calculator:	A calculator that will store and draw the graphs of several functions at once.
Independent events:	Events such that the outcome of the first event has no effect on the probabilities of the outcome of the second event (e.g., two tosses of the same coin are independent events).
Inductive reasoning:	Generalizations made from particular observations in a common occurrence.
Inequality:	A mathematical sentence that contains a symbol (e.g., $>, <, \geq, \leq$ or $\neq$) in which the terms on either side of the symbol are unequal (e.g., $x < y$, $7 > 3$, $n \geq 4$).
Infinite:	Has no end or goes on forever.
Integer:	A number that is a positive whole number, a negative whole number or zero.
Inverse:	A new conditional formed by negating both the antecedent and the consequent of a conditional.
Inverse operations:	Operations that undo each other (e.g., addition and subtraction are inverse operations; multiplication and division are inverse operations).

Academic Standards for Mathematics

Inverse variation: When the ratio of one variable to the reciprocal of the other is constant, one of them is said to vary inversely as the other.

Irrational number: A number that cannot be written as a simple fraction. It is an infinite and non-repeating decimal.

Limit: A number to which the terms of a sequence get closer so that beyond a certain term all terms are as close as desired to that number.

Line of best fit: The line that fits a set of data points with the smallest value for the sum of the squares of the errors (vertical distances) from the data points to the line; the regression line.

Linear function: A function whose general equation is $y = mx + b$, where m and b stand for constants and $m \neq 0$.

Linear measurement: Measurement in a straight line.

Logarithm: The exponent indicating the power to which a fixed number, the base, must be raised to produce a given number. For example, if $n^x = a$, the logarithm of a, with n as the base, is x; symbolically, $\log_n a = x$. If the base is 10, the log of 100 is 2.

Manipulatives: Materials that allow students to explore mathematical concepts in a concrete mode.

Mathematical model: A representation in the mathematical world of some phenomenon in the real world. It frequently consists of a function or relation specifying how two variables are related.

Matrix: A rectangular array of numbers representing such things as the coefficients in a system of equations arranged in rows and columns.

Maximum: The greatest number in a set of data.

Mean: The sum of the set of numbers divided by n, the number of numbers in the set.

Academic Standards for Mathematics

Median: The number that lies in the middle when a set of numbers is arranged in order. If there are two middle values, the median is the mean of these values.

Metric system: A system of measurement used throughout the world based on factors of 10. It includes measures of length, weight and capacity.

Minimum: The least number in a set of data.

Missing addend: A member of an addition number sentence in which that term is missing (e.g., 5 + __ = 8).

Mode: The number(s) that occurs most often in a set of numbers (e.g., in the set 1, 2, 3, 3, 5, 8; the mode is 3).

Multiple: A number that is the product of a given integer and another integer (e.g., 6 and 9 are multiples of 3).

Normal curve: A graphical plot of a mathematical function (frequency distribution) which is unimodal and symmetrical.

One-to-one correspondence: When one and only one element of a second set is assigned to an element of a first set, all elements of the second set are assigned, and every element of the first set has an assignment, the mapping is called one-to-one (e.g., in the set Bill Clinton, George Bush, Ronald Reagan, Jimmy Carter, Hillary Clinton, Barbara Bush, Nancy Reagan and Rosalynn Carter, there is a one-to-one correspondence between the pairs).

Open sentence: A statement that contains at least one unknown. It becomes true or false when a quantity is substituted for the unknown (e.g., x + 5 = 9, y - 2 = 7).

Order of operations: Rules for evaluating an expression: work first within parentheses; then calculate all powers, from left to right; then do multiplications or divisions, from left to right; then do additions and subtractions, from left to right.

Academic Standards for Mathematics

Patterns: Regularities in situations such as those in nature, events, shapes, designs and sets of numbers (e.g., spirals on pineapples, geometric designs in quilts, the number sequence 3, 6, 9, 12,…).

Permutation: An arrangement of a given number of objects from a given set in which the order of the objects is significant.

Perpendicular lines: Two lines that intersect to form right angles (e.g., ⊥, ↵, ⌜).

Plotting points: Locating points by means of coordinates, or a curve by plotted points, representing an equation by means of a curve so constructed.

Polygon: A union of segments connected end to end, such that each segment intersects exactly two others at its endpoints.

Powers: A number expressed using an exponent. The number 5^3 is read five to the third power or five cubed.

Prime: An integer greater than one whose only positive factors are 1 and itself (e.g., 2, 3, 5, 7, 11, 13, 17, and 19).

Probability: A number from 0 to 1 that indicates how likely something is to happen.

Problem-solving: Finding ways to reach a goal when no routine path is apparent.

Proof by contradiction: A proof in which, if s is to be proven, one reasons from not-s until a contradiction is deduced; from this it is concluded that not-s is false, which means that s is true.

Proportion: An equation of the form $\frac{a}{b} = \frac{c}{d}$ that states that the two ratios are equivalent.

Quadrilateral: A four-sided polygon.

Quartiles: The three values that divide an ordered set into four subsets of approximately equal size. The second quartile is the median.

Radian: A unit of angular measure equal to $\frac{1}{2\pi}$ of a complete revolution.

Range (1) : The difference between the greatest number and the least number in a set of data.

Range (2) : The set of output values for a function.

Rate of change: The limit of the ratio of an increment of the function value at the point to that of the independent variable as the increment of the variable approaches zero.

Ratio: A comparison of two numbers by division.

Rational numbers: Any number that can be written in the form $\frac{a}{b}$ where a is any integer and b is any integer except zero.

Real numbers: The set consisting of all rational numbers and all irrational numbers.

Reasonableness: Quality of a solution such that it is not extreme or excessive.

Reciprocal: The fractional number that results from dividing one by the number.

Rectangular prism: A three-dimensional figure whose sides are all rectangles; a box.

Academic Standards for Mathematics

Reflection: A transformation that produces the mirror image of a geometric figure.

Regression: The line that represents the least deviation from the points in a scatter plot of data.

Regular polygon: A polygon in which all sides have the same measure and all angles have the same measure.

Relation: A set of ordered pairs.

Reliability: The extent to which a measuring procedure yields the same results on repeated trials.

Repeated addition: A model for multiplication (e.g., $2 + 2 + 2 = 3 \times 2$).

Rotation: A transformation that maps every point in the plane to its image by rotating the plane around a fixed point or line.

Scientific calculator: A calculator that represents very large or very small numbers in scientific notation with the powering, factorial, square root, negative and reciprocal keys.

Scientific notation: A way in writing a number of terms of an integer power of 10 multiplied by a number greater than or equal to 1 and less than 10.

Sequence: A set of ordered quantities (e.g., positive integers).

Series: The indicated sum of the terms of a sequence.

Similarity: Having the same shape but not necessarily the same size.

Simple event: An event whose probability can be obtained from consideration of a single occurrence (e.g., the tossing of a coin is a simple event).

Academic Standards for Mathematics

Simulation: Modeling a real event without actually observing the event.

Slope: The slope of a line is the ratio of the change in y to the corresponding change in x; the constant m in the linear function equation; rise/run.

Standard deviation: The square root of the variance.

Stem-and-leaf plot: A frequency distribution made by arranging data (e.g., student scores on a test were 98, 96, 85, 93, 83, 87, 85, 87, 93, 75, 77 and 83). This data is displayed in a stem-and-leaf plot below.

9	8, 6, 3, 3
8	7, 7, 5, 5, 3, 3
7	7, 5

Systems of equations: Two or more equations that are conditions imposed simultaneously on all the variables, but may or may not have common solutions (e.g., $x + y = 2$, and $3x + 2y = 5$).

Symmetry: A line of symmetry separates a figure into two congruent halves, each of which is a reflection of the other (e.g., ∅, the line through the center of the circle divides it into congruent halves).

t-test: A statistical test done to test the difference of means of two samples.

Tessellation: A repetitive pattern of polygons that covers an area with no holes and no overlaps (e.g., floor tiles).

Transformation: An operation on a geometric figure by which each point gives rise to a unique image.

Translation: A transformation that moves a geometric figure by sliding each of the points the same distance in the same direction.

Academic Standards for Mathematics

Tree diagram: A diagram used to show the total number of possible outcomes in a probability experiment.

Trigonometric functions: A function (e.g., sine, cosine, tangent, cotangent, secant, cosecant) whose independent variable is an angle measure, usually in degrees or radians.

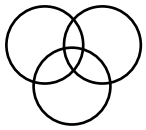
Valid argument: An argument with the property that no matter what statements are substituted in the premises, the truth value of the form is true. If the premises are true, then the conclusion is true.

Variable: A symbol used to stand for any one of a given set of numbers or other objects (e.g., in the equation $y = x + 5$, y and x are variables).

Variance: In a data set, the sum of the squared deviations divided by one less than the number of elements in the set (sample variance s^2) or by the number of elements in the set (population variance σ^2).

Vector: A quantity that has both magnitude and direction (e.g., physical quantities such as velocity and force).

Venn diagram: A display that pictures unions and intersections of sets.



Volume: The amount of space enclosed in a space (3-dimensional) figure, measured in cubic units.

Y-intercept: The y-intercept of a line is the y-coordinate of the point at which the graph of an equation crosses the y-axis.

π: pi, the ratio of the circumference of a circle to its diameter: 3.1415926535.

Academic Standards for Mathematics